

MA 3012

Polytechnic University
MIDTERM FEBRUARY 16, 2007

Print Name:

Signature:

ID #:

Instructor/Section:

Directions: Answer all questions. To obtain full credit, you must show your reasoning in all your answers, including any rules or formulae you rely on. Full marks will only be awarded for answers which have been simplified to provide exact solutions(e.g. of the form $1/19$ and not 0.052). The last page contains formulae that you may find helpful. You may tear that page out.

If you are feeling ill you should inform the proctor. The proctor will note your name, Poly ID and accept any written statement(s) that you may wish to make regarding your illness.

Problem	Possible	Points
1	15	
2	16	
3	18	
4	18	
5	15	
6	18	
7*	5	
Total	100+5	

*: This problem is optional and gives extra credit

YOUR SIGNATURE:

(1) (Chapter 1,2) If a student grabs (with his/her eyes closed) two socks from his/her drawer containing 8 white socks, 3 blue socks, and 11 black socks, what is the probability that he/she gets

(a) a white pair?

(b) no black sock among the pair?

(c) no matching pair?

YOUR SIGNATURE:

- (2) (Chapter 2,3) A box contains three coins. One of the coins has two heads, while the other two coins are normal and unbiased. A coin is chosen at random from the box and tossed four times. It is reported to you that a head was recorded each time this coin was tossed. What is the probability that the coin is the two-headed one?

YOUR SIGNATURE:

(3) (Chapter 2,3) Lee v.C and Clint E. are involved in a duel. The rules are the following. They pick up their guns and shoot at each other simultaneously. If both shots miss, they repeat the process. If at least one of them is dead, the duel ends. Suppose that their shooting skills are independent and that Clint is hit(=dies) with probability $1/2$ and that Lee is hit(=dies) with probability $2/3$. Find the probability that

(a) Clint survives round one whereas Lee dies in round one.

(b) Clint and Lee both die in round n (not before).

(c) Clint survives the duel (infinitely many rounds).

YOUR SIGNATURE:

(4) (Chapter 3) Let A and B be two events with $P(A) = 0.8$, $P(B|A) = 0.5$, and assume that A and B are independent. Show for each of the following statements that it is true or show that it is false.

(a) A and B are mutually exclusive.

(b) A and $A \cap B$ are independent.

(c) $P(B) = \frac{1}{2}$.

YOUR SIGNATURE:

(5) (Chapter 4) The computer of the dating agency *lonely-hearts.com* evaluates the relationships of the couples formed through its matching algorithm in the age category 20-30. A relationship is counted as a success if it holds for more than 5 years. It turns out that 70% of the couples in this category are successes. The computer algorithm creates 10 new couples. What is the probability that

(a) Exactly 5 of them will be success?

(b) More than 8 will be successes?

(c) What is the expected number of successes?

YOUR SIGNATURE:

(6) (Chapter 4) A certain instructor had $n > 1$ keys of which only one fits her office door. The keys have become completely scrambled on her key chain, so that she randomly tries them until finding the key that works. She is not (yet) in a state where she tries a given key more than once! Thus, she may find the correct key on the first try, the second try, or the worst case, at the n th try.

(a) What is the probability of each of these outcomes? That is, determine the probability mass function for the event $P(X = r)$ that the correct key is found at exactly the r th trial.

(b) Determine the expected number of tries she needs.

(c) Compute the variance of X .

YOUR SIGNATURE:

- (7) (**Extra Credit**) A student is trying to copy from his/her neighbor's Math exam. Since he/she cannot see well enough, he/she makes transcription errors which he/she doesn't recognize due to his/her scanty knowledge of the material. Assume that the number of errors per page is Poisson distributed with parameter $\lambda = 3$. Determine the probability that he/she can copy two pages without a single error.