Polytechnic University

MA 3012 FINAL MARCH 9, 2007

Print Name:

Signature:

ID #:

Instructor/Section:

Directions: Answer all questions. To obtain full credit, you must show your reasoning in all your answers, including any rules or formulae you rely on. Full marks will only be awarder for answers which have been simplified to provide exact solutions(e.g. of the form 1/19 and not 0.052). The last page contains formulae that you may find helpful. You may tear that page out.

If you are feeling ill you should inform the proctor. The proctor will note your name, Poly ID and accept any written statement(s) that you may wish to make regarding your illness.

Problem	Possible	Points
1	20	
2	18	
3	12	
4	14	
5	18	
6	18	
7*	5	
Total	100+5	

*: This problem is optional and gives extra credit

YOUR SIGNATURE:

(1) (Chapters 3,4,6) A friend offers you to play the following game of chance. The rules are to roll a fair die three times. If he gets the number 3 on one of the rolls, you will pay him \$10; If he does not get the 3 on any of them, he will pay you \$5.

(a) Let X be the random variable that represents your gain. Find the probably mass function of X.

X		
P(X=x)		

(b) Since you find out from (a) that the expected value is to your disadvantage, you propose him to change the rules of the game as follows: The numbers from the three rolls are added. If the sum is less than or equal to 6, you will pay him \$112, or otherwise, he will pay you \$11.
Let Y be the random variable that represents your gain. Find the probability mass function of Y.

Y		
P(Y=y)		

(c) As your friend feels ar a disadvantage with the second game, you settle on playing both at the same time. Determine the joint probability mass function of X and Y.

X,Y			Sum
Sum			

(d) Determine E(X+Y) and check whether the game is fair.

(e) Are X and Y independent?

(2) (Chapter 5)

(a) A bottling company observes that the amount X of beer which is automatically filled into bottles is distributed according to $N(0.7, 0.01^2)$, i.e., where $\mu = 0.7$ liters are $\sigma = 0.01$ liters. What is the probability that there are at least 0.71 liters in a randomly chosen bottle?

(b) A box manufactured to contain 12 of the beer bottles from (a) weights 500 grams when it is empty. Determine the expected weight of a full box (assuming that 1 liter of beer weight 1000 grams).

(c) The company also sells a rare brand of cognac. For the amount Y of cognac, it has been established that $\mu = 0.7$ liters, but the variance σ^2 is unknown. The company wishes to fine- tune he bottling process, so that 95% of the amount of cognac filled into each bottle lies between 0.69 liters and 0.71 liters. determine the variance that Y must have in order to achieve this goal.

(3) (Chapter 3,4,6) A bakery uses 600 raisins for the production of 100 heart-healthy raisin-almond bagels. The distribution of raisins in the bagels follows a Poisson process $P(\lambda_1)$.

(a) Determine the probability that in a randomly chosen bagel there are more than 3 raisins.

(b) Assume that the number Y of almonds on the bagels is distributed independently from the raising according to a Poisson distribution with parameter $\lambda_2 = 2$. Determine the probability that the number of raisins and almond together in a randomly chosen bagel is smaller than 4. (i.e., Find $P(X+Y<4)$.

(4) (Chapter 6) The random variables X and Y have the joint density

$$f_{X,Y}(x,y) = \begin{cases} 2, & 0 \le x \le 1, 0 \le y \le x \\ 0, & \text{otherwise.} \end{cases}$$

(a) Sketch the set $\{(x,y) \in \mathbb{R}^2 : f_{X,Y}(x,y) > 0\}$.

(b) Determine the marginal density $f_X(x)$ of X and the marginal density $f_Y(y)$ of Y. (Hint: The result is $f_X(x) = 2x$ for $0 < x < 1$, $f_Y(y) = 2 - 2y$ for $0 < y < 1$)

(c) Compute $E(X)$ and $E(Y)$.

(d) Are X and Y independent (show all work!)?

(e) Compute $P(Y \ge \frac{1}{2} \mid \frac{1}{2} < X < 1))$ and verify your result by indicating the corresponding area in your sketch from (a).

(5) (Chapter 6) The random variables X and Y are independently distributed, both according to an exponential distribution with parameters $\lambda = 9$.

(a) Compute the cumulative distribution function $F_Z(z)$ for $Z = X + Y$.

(b) Find $f_Z(z)$.

(c) Find the probability $P(Z > \frac{1}{9})$.

YOUR SIGNATURE: ______________________

(6) (Chapter 5) The measured radius X in millimeters of a circular raindrop on the pavement is a continuous random variable with density function given by

$$f_X(x) = 6x(1-x), \quad 0 < x < 1,$$

and zero otherwise.

(a) Find the density function $f_Y(y)$ of the area $Y = \pi X^2$ of the raindrop.

(b) Verify that the result is indeed a probability density function.

(c) Find the probability that the raindrop has an area smaller than $\frac{\pi}{4}$ square millimeters.

(7) (Extra Credit) At a party, there is a heated discussion about whether tall men prefer tall women as their girl-friends and conversely, or not. Let X denote the body height of the women, and Y the body height of the men.

(a) At the party there already four couples present. Their respective body heights in meters are given in the table below.

Couple ω	1	2	3	4
$X(\omega)$	1.70	1.60	1.70	1.40
$Y(\omega)$	1.90	1.90	1.70	1.70

Determine the covariance of X and Y.

(b) Based on the given data, what is your opinion in the discussion at the party?