Directions: You have 55 minutes to answer the following questions. You must show all your work as neatly and clearly as possible and indicate the final answer clearly. You may use a calculator.

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(1) (15 points) A box of 40 components is called acceptable if it contains no more than 2 defectives. The procedure for sampling the box is to select 5 components at random and to reject the box if one or more defectives are found.

What is the probability that a box with 3 defectives (not acceptable if you inspect the entire box) will be accepted if we use the above sampling procedure?
(2) (15 points) A coin is biased so that a tail is three times as likely to occur as a heads.

Consider a game that you need to pay a dollar to play. In the game, you are asked to toss this coin, if a tail occurs, you lose your entry fee of $1; if a heads occurs, you will be paid 3 dollars (you don’t get your initial fee back either.)

(a) Let \( X \) represent your net winning of this game, write out the probability mass function (p.m.f) of \( X \).

(b) If you play this game 10 times, what is your expected winning?
(3) (40 points) An oil drilling company ventures into various locations, and their success or failure is independent from one location to another. Suppose the probability of a success at any specific location is 0.3.

(a) What is the probability that a driller drills 10 locations and finds 1 success?

(b) What is the expected number of drills it takes to get the first success?

(c) The driller feels that he will go bankrupt if it will take make than 10 drills to get the first success. What is the driller’s probability of bankruptcy?

(d) The driller feels that he will “hit it big” if the third success occurs on or before the fifth attempt. What is the probability that the driller will “hit it big”? 
(4) (15 points) Define the p.m.f. and give the values of $\mu$, $\sigma^2$, and $\sigma$ when the moment generating function of $X$ is defined by

$$M(t) = 0.4 + 0.6e^t.$$
(5) (15 points) Urn A contains 2 white balls and 1 black ball, whereas urn B contains 1 white ball and 5 black balls. A ball is drawn at random from urn A and placed in urn B. A ball is then drawn from urn B.

(a) What is the probability that the ball drawn from urn B after the transferring is white?
(b) Given that the ball drawn from B was indeed white, what is the conditional probability that the ball transferred was white?
Some definitions and formulas you might find useful

(1) For a random sample \( x_1, x_2, \ldots, x_n \), the sample mean and the sample variance are defined as
\[
\bar{x} = \frac{1}{n} \sum_{i=1}^{n} x_i \quad \text{and} \quad s^2 = \frac{1}{n-1} \sum_{i=1}^{n} (x_i - \bar{x})^2.
\]

(2) For a random sample \( x_1, x_2, \ldots, x_n \), when the observations are ordered from the smallest to the largest, the resulting ordered data 
\[ x_{(1)} \leq x_{(2)} \leq \ldots \leq x_{(n)} \]
are called the order statistics.

(3) If \( 0 < p < 1 \), the \((100p)\)th sample percentile of \( x_1, x_2, \ldots, x_n \) has approximately \( np \) sample observations less than it and also \( n(1-p) \) sample observations greater than it. If \( (n+1)p \) is an integer \( r \) plus some proper fraction \( a/b \), one way to define the \((100p)\)th sample percentile \( \tilde{\pi}_p \) is
\[
\tilde{\pi}_p = x_{(r)} + \frac{a}{b} (x_{(r+1)} - x_{(r)}) = (1 - \frac{a}{b}) x_{(r)} + \frac{a}{b} x_{(r+1)},
\]
where \( x_{(r)} \) and \( x_{(r+1)} \) are the \( r \)th and \((r+1)\)th order statistics.

(4) \( P(n, r) = \frac{n!}{(n-r)!} \); \( C(n, r) = \binom{n}{r} \) where \( \binom{n}{r} = \frac{n!}{r!(n-r)!} \).

(5) The conditional probability of \( A \), given \( B \), is \( P(A|B) = \frac{P(A \cap B)}{P(B)} \), if \( P(B) > 0 \).

(6) (Bayes’ Theorem) If the events \( B_1, B_2, \ldots, B_k \) constitute a partition of the sample space \( S \), where \( P(B_i) > 0 \) for \( i = 1, 2, \ldots, k \), then for any event \( A \) in \( S \) such that \( P(A) > 0 \),
\[
P(B_r|A) = \frac{P(B_r \cap A)}{\sum_{i=1}^{k} P(B_i \cap A)} = \frac{P(B_r)P(A|B_r)}{\sum_{i=1}^{k} P(B_i)P(A|B_i)} \quad \text{for} \quad r = 1, 2, \ldots, k.
\]

(7) The cumulative distribution function \( F(x) \) of a random variable \( X \) with p.d.f. \( f(x) \) is \( F(x) = P(X \leq x) \).

(8) Assume that \( X \) is a continuous random variable with p.d.f. \( f(x) \). Then the expectation and the variance \( X \) are defined as,
\[
E(X) = \int_{-\infty}^{\infty} xf(x) \, dx; \quad \text{and}, \quad Var(X) = E[(X - E(X))^2] = E(X^2) - (E(X))^2.
\]

The moment generating function of \( X \), if it exists, is defined as
\[
M(t) = E(e^{tX}) = \int_{-\infty}^{\infty} e^{tx} f(x) \, dx \quad -h < t < h.
\]
If $X$ is a discrete random variable, just replace all the integrals with appropriate summations.

(9) The $r$th moment of a random variable $X$ is defined as $\mu_r = E(X^r)$.

(10) The p.d.f. for Poisson distribution is $f(x) = \frac{e^{-\lambda} \lambda^x}{x!}$, for $x = 0, 1, 2, \ldots$. The mean and variance of this distribution are $\lambda$ and $\lambda$ respectively.

(11) Integration by Parts:
$$\int u dv = uv - \int v du \quad \text{or} \quad \int uv' dx = uv - \int vu' dx$$

(12) The $n$th degree Taylor Polynomial of $f(x)$ centered at $x = a$:
$$f(x) = f(a) + f'(a)(x-a) + \frac{f''(a)}{2!}(x-a)^2 + \frac{f'''(a)}{3!}(x-a)^3 + \cdots + \frac{f^{(n)}(a)}{n!}(x-a)^n$$

(13) Taylor series of $f(x)$ centered at $x = a$:
$$f(x) = f(a) + f'(a)(x-a) + \frac{f''(a)}{2!}(x-a)^2 + \frac{f'''(a)}{3!}(x-a)^3 + \cdots$$

(14) Taylor Series of important functions:
$$\sin(x) = x - \frac{x^3}{3!} + \frac{x^5}{5!} - \frac{x^7}{7!} + \cdots$$
$$\cos(x) = 1 - \frac{x^2}{2!} + \frac{x^4}{4!} - \frac{x^6}{6!} + \cdots$$
$$e^x = 1 + x + \frac{x^2}{2!} + \frac{x^3}{3!} + \frac{x^4}{4!} + \cdots$$
$$\frac{1}{1-x} = 1 + x + x^2 + x^3 + \cdots \quad \text{for} \quad -1 < x < 1$$
$$\ln(1+x) = x - \frac{x^2}{2} + \frac{x^3}{3} - \frac{x^4}{4} + \cdots \quad \text{for} \quad -1 < x \leq 1$$
$$\left(1 + x\right)^p = 1 + px + \frac{p(p-1)}{2!}x^2 + \frac{p(p-1)(p-2)}{3!}x^3 + \cdots \quad \text{for} \quad -1 < x < 1$$

(15) Finite Geometric Sum:
$$a + ax + ax^2 + \cdots + ax^{n-1} = \frac{a(1-x^n)}{1-x}$$

(16) Infinite Geometric Series:
$$a + ax + ax^2 + \cdots = \frac{a}{1-x} \quad \text{for} \quad |x| < 1$$