

Polytechnic University

MA 2112

MIDTERM

MAY 23, 2003

Print Name:

Signature:

ID #:

Instructor/Section:

Directions: You have 90 minutes to answer the following questions. You must show all your work as clearly as possible. You may use a calculator. The last page contains a list of useful formulas. You may tear that page out.

Problem	Possible	Points
1	20	
2	10	
3	10	
4	10	
5	15	
6	20	
7	15	
Total	100	

(1) Fill in the blanks. (You do not have to explain.)

(a) The surface $z = e^{x^2+y^2} - 1$ intersects the x -axis at the point(s) _____

(b) In 3-space the surface $x^2 + z^2 = 9$ represents

(c) If f is a linear function of two variables such that $f(0, 0) = 1$, $f(0, 1) = 2$ and $f(1, 0) = -1$, then $f(1, 1) =$ _____

(d) The domain of the function $f(x, y, z) = \ln(16 - x^2 - y^2 - z^2)$ is

(e) The contours of $f(x, y) = -2x + y - 3$ are lines with slope(s) equal to _____

(2) (Worksheet I) Consider the surface

$$z = 4(x - 2)^2 - y^2.$$

(a) Sketch the cross section of the surface by the plane $y = 0$.

(b) Sketch the level curve(s) corresponding to $z = 0$.

- (3) (Problem 10, Page 595) Find a formula for a function $w = f(x, y, z)$ whose level surface $w = 4$ is a sphere of radius 2, centered at the point $(-2, 0, 1)$.

- (4) (Worksheet II) Find the equation of the linear function whose graph intersects the xz -plane along the line $x + z = 1$ and intersects the yz -plane along the line $y - z = -1$.

- (5) (Problem 26, Page 627) Find the points where the plane $z = 5x - 4y + 3$ intersects each of the coordinate axes. Find the lengths of the sides and the angles of the triangle formed by these points.

(6) Consider the vector $\vec{a} = 5\vec{i} + \vec{j} - 4\vec{k}$.

Fill in the blanks. (You do not have to explain.)

(a) If \vec{a} is the position vector of point P , then $P =$ _____

(b) A unit vector parallel to \vec{a} is _____

(c) A vector perpendicular to \vec{a} is _____

(d) If \vec{b} is a vector parallel to \vec{a} such that $\vec{a} \cdot \vec{b} = -2$, then $\|\vec{b}\| =$ _____

(e) The angle between the plane $4(x - 1) + y + 5z = 0$ and a plane perpendicular to \vec{a} is _____

- (7) Find a , b and c such that the plane $ax+by+z = c$ is parallel to the plane $z = 2x-y+2$ and the distance between the two planes is equal to 6.

Useful Formulas

- The **dot product** of two vectors:

$$\vec{v} \cdot \vec{w} = \|\vec{u}\| \|\vec{w}\| \cos \theta = v_1 w_1 + v_2 w_2 + v_3 w_3.$$

- The **cross product** of two vectors:

$$\vec{v} \times \vec{w} = (\|\vec{u}\| \|\vec{w}\| \sin \theta) \vec{n} = (v_2 w_3 - w_3 v_2) \vec{i} + (v_3 w_1 - v_1 w_3) \vec{j} + (v_1 w_2 - v_2 w_1) \vec{k}.$$

- **Volume of a parallelepiped** with edges \vec{a} , \vec{b} , \vec{c} : Volume = $|(\vec{b} \times \vec{c}) \cdot \vec{a}|$.