

MA 2012

Polytechnic University
SAMPLE MIDTERM

SEPTEMBER, 2006

Print Name:

Signature:

ID #:

Instructor:

Directions: You have 90 minutes to answer the following questions. **You must show all your work** as neatly and clearly as possible and indicate the final answer clearly. You may use a calculator.

When you use the Gaussian algorithm explain each step by a remark like “adding -4 times the first row to the third row” or $R_3 = R_3 - 4R_1$.

If you use a computation rule, like $(AB)^T = B^T A^T$, state the rule itself.

If you are feeling ill you should inform the proctor. The proctor will note your name, Poly ID and accept any written statement(s) that you may wish to make regarding your illness.

Problem	Possible	Points
1	12	
2	18	
3	10	
4	20	
5	15	
6	15	
7	10	
Total	100	

YOUR SIGNATURE:

(1) Determine whether each of the following statements is TRUE or FALSE. You do not need to explain.

(a) If $A = A^{-1}$, then $\det(A)$ must either equal to 1 or -1 .

(b) If $\operatorname{tr}(A) = k$, then $\operatorname{tr}(A^n) = k^n$.

(c) If A and B are two $n \times n$ matrices, then $(A + B)^2 = A^2 + 2AB + B^2$.

(d) If A and B are two $n \times n$ matrices such that $AB = 0$, then either $A = 0$ or $B = 0$.

(e) If A and B are two $n \times n$ matrices, then $(AB)^2 = B^2A^2$.

(f) If A and B are two $n \times n$ matrices, then $\det((AB)^T) = \det(A) \det(B)$.

YOUR SIGNATURE:

(2) (a) Let

$$M = \begin{pmatrix} 3+i & 4 \\ -i & 3-i \end{pmatrix}, \quad N = \begin{pmatrix} 2i & 1+i \\ 1-i & -2i \end{pmatrix}.$$

Find the trace of NMN^{-1} .

(b) If $A^{-1}BC^T$ is a 5×7 matrix and $B^T A^{-1}$ is a 6×5 matrix, what are the dimensions of the three matrices A , B and C ?

(c) Let A be a 4×4 matrix with $\det(A) = -2$ and $\text{tr}(A) = 3$, and let B be a 4×4 matrix with $\det(B) = 16$ and $\text{tr}(B) = 5$. Evaluate each of the following expressions, or state if there is not enough information to evaluate.

(i) $\det(2A^2B^{-1}) =$ _____

(ii) $\text{tr}(2B^T - 3I) =$ _____

YOUR SIGNATURE:

(3) Let

$$A = \begin{pmatrix} -1 & 2 & 4 & -1 \\ 0 & 1 & 3 & 4 \\ 0 & 1 & 0 & 1 \\ 2 & 1 & -5 & 3 \end{pmatrix}.$$

(a) Find $A + A^T$ and $A - A^T$.

(b) Express A as $B + C$, where B is symmetric and C is skew symmetric.

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(4) Let

$$A = \begin{pmatrix} 1 & 1/2 & 1/3 \\ 1/2 & 1/3 & 1/4 \\ 1/3 & 1/4 & 1/5 \end{pmatrix}.$$

(a) Show that A is invertible. Find A^{-1} .

(b) Find a matrix X such that

$$\begin{pmatrix} 1 & 1/2 & 1/3 \\ 1/2 & 1/3 & 1/4 \\ 1/3 & 1/4 & 1/5 \end{pmatrix} X = \begin{pmatrix} 3 & -1 & 2 & 0 \\ 2 & 0 & 1 & 5 \\ 0 & 2 & 4 & 1 \end{pmatrix}.$$

YOUR SIGNATURE:

(5) Determine the values of k for which the system

$$x - y - z = 2$$

$$3x - 2y + kz = k + 8$$

$$4x + (1 - k)y + 3z = 8$$

in unknowns x , y and z over \mathbb{C} has

- (a) a unique solution,
- (b) no solutions,
- (c) infinitely many solutions.

YOUR SIGNATURE:

(6) Short proofs. Carefully show your explanation.

(a) Let A and B be two invertible $n \times n$ matrices such that $A^2 = I$ and $B^2 = I$. Show that $(AB)^{-1} = BA$.

(b) Let S be an $n \times n$ skew-symmetric matrix, and let T be an $n \times n$ orthogonal matrix. Is $T^{-1}S^2T$ symmetric?

YOUR SIGNATURE:

- (7) A tour bus load of 45 people attended two Florida theme parks on successive days. On the first day, the entrance fee was \$15 per adult, \$8 per child, \$12 per senior citizen and the total charge was \$558. On the second day, the entrance fee was \$20 per adult, \$12 per child, \$17 per senior citizen and the total charge was \$771. How many adults, children and senior citizens were on this tour bus?