

MA 2012

Polytechnic University

FINAL

AUGUST 23, 2007

Print Name:

Signature:

ID #:

Instructor:

Directions: You have **90 minutes** to answer the following questions. **You must show all your work** as neatly and clearly as possible and indicate the final answer clearly. You may use a calculator.

When you use the Gaussian algorithm explain each step by a remark like “adding -4 times the first row to the third row” or $-4R_1 + R_3$.

If you use a computation rule, like $(AB)^T = B^T A^T$, state the rule itself.

If you are feeling ill you should inform the proctor. The proctor will note your name, Poly ID and accept any written statement(s) that you may wish to make regarding your illness.

Problem	Possible	Points
1	15	
2	20	
3	15	
4	15	
5	15	
6	20	
Total	100	

YOUR SIGNATURE:

(1) (15 points) Let A and B be $n \times n$ invertible matrices and k be a non zero real number. Determine whether each of the following statements is TRUE or FALSE. You do not have to explain.

(a) $(AB)^{-1} = A^{-1}B^{-1}$.

(b) $(kA)^{-1} = k^{-1}A^{-1}$.

(c) $\det(A - B) = \det(A) - \det(B)$.

(d) $\det(AB) = \det(A)\det(B)$.

(e) $\det(kA) = k^n \det A$.

YOUR SIGNATURE:

(2) (20 points) Consider the set of vectors in the real vector space \mathbb{R}^3 :

$$\left\{ \begin{pmatrix} 1 \\ 3 \\ 3 \end{pmatrix}, \begin{pmatrix} 0 \\ 1 \\ -p \end{pmatrix}, \begin{pmatrix} 1 \\ p \\ -1 \end{pmatrix} \right\}$$

For what value(s) p is the set of vectors linearly independent? Show all your work.

YOUR SIGNATURE:

(3) Consider the matrix

$$A = \begin{pmatrix} 1 & 2 & 1 & 3 & 2 \\ 3 & 4 & 9 & 0 & 7 \\ 2 & 3 & 5 & 1 & 8 \\ 2 & 2 & 8 & -3 & 5 \end{pmatrix}.$$

(a) (4 Points) Find a basis for the column space of A . You do not need to show work.

(b) (5 Points) Find a basis for the null space of A . You do not need to show work.

(c) (2 Points) Find the rank of A . You do not need to show work.

(d) (2 Points) Find the nullity of A . You do not need to show work.

(e) (2 Points) Find the rank of A^T . You do not need to show work.

YOUR SIGNATURE:

(4) (15 points) Determine whether each of the following statements is TRUE or FALSE. You do not have to explain.

(a) If $W = \left\{ \begin{pmatrix} a \\ b \end{pmatrix} \mid a > b, \ a, b \in \mathbb{R} \right\}$, then W is a subspace of the real vector space \mathbb{R}^2 .

(b) If $W = \left\{ \begin{pmatrix} a \\ b \end{pmatrix} \mid a \geq b, \ a, b \in \mathbb{R} \right\}$, then W is a subspace of the real vector space \mathbb{R}^2 .

(c) If $W = \left\{ \begin{pmatrix} a \\ b \end{pmatrix} \mid a = b, \ a, b \in \mathbb{R} \right\}$, then W is a subspace of the real vector space \mathbb{R}^2 .

YOUR SIGNATURE:

- (5) For the real vector spaces \mathbb{R}^3 and \mathbb{R}^2 , consider the transformation $T : \mathbb{R}^3 \rightarrow \mathbb{R}^2$ defined by

$$T \begin{pmatrix} a \\ b \\ c \end{pmatrix} = \begin{pmatrix} 3a + 2b \\ 4b - c \end{pmatrix}.$$

- (a) (10 Points) Show that T is linear using the definition of a linear transformation.

- (b) (3 Points) Find the standard matrix representation for T .

- (c) (2 Points) Does the inverse of T exist? You do not need to explain.

YOUR SIGNATURE:

(6) Consider the following matrix:

$$A = \begin{pmatrix} 4 & 2 \\ 3 & -1 \end{pmatrix}.$$

(a) (10 Points) Find the eigenvalues of A . Show all your work.

(b) (10 Points) For each eigenvalues of A find its corresponding eigenspace, and give a basis for each eigenspace.