Polytechnic University

MA 2012 FINAL JANUARY 16, 2007

Print Name:

Signature:

ID #:

Instructor:

Directions: You have **90 minutes** to answer the following questions. **You must show all your work** as neatly and clearly as possible and indicate the final answer clearly. You may use a calculator.

When you use the Gaussian algorithm explain each step by a remark like "adding -4 times the first row to the third row" or $R_3 \to R_3 - 4R_1$.

If you use a computation rule, like $(AB)^T = B^T A^T$, state the rule itself.

Problem	Possible	Points
1	15	
2	15	
3	15	
4	10	
5	15	
6	15	
7	15	
8* Bonus	6	
Total	106	

(1) Determine whether each of the following statements is TRUE or FALSE. (You do not have to explain.)

(a) The row space and the null space of any matrix A have the same dimension.

(b) Let A be an invertible $n \times n$ matrix. The linear system $A^{-1}\vec{x} = \vec{b}$ has a unique solution for every vector $\vec{b} \in \mathbf{R}^n$.

(c) If A, B and C are two $n \times n$ matrices such that $AB{=}AC$, then $B{=}C$.

(d) If $\lambda = 0$ is an eigenvalue of a matrix A, then A is not diagonalizable.

(e) If A and B ate two nonsingular $n \times n$ matrices, then the matrix AB is also nonsingular.

(2) (a) The 2×2 matrices A and B satisfy $\det(2A) = 4$, and $\det(B^3)$=27. Find the $\det((AB)^{-1})$.

(b) An 5×5 matrix A has characteristic equation $(\lambda - 1)^3(\lambda - 2)(\lambda - 3) = 0$. Find:
(i) $\det(A)$

(ii) $\mathrm{tr}(A)$

(3) In which of the following parts, decide whether V_1 and V_2 are subspaces of the vector space of the set of all 2×2 real matrices. Show all of your work.

(a)

$$V_1 = \left\{ \begin{pmatrix} a & b \\ c & d \end{pmatrix}; a, b, c, d \in \mathbf{R} : \ a + d = 0 \right\}.$$

(b)

$$V_2 = \left\{ \begin{pmatrix} a & b \\ c & d \end{pmatrix}; a, b, c, d \in \mathbf{R} : \ ad = 0 \right\}.$$

(4) Fill in the blanks. You do not have o give explanations.

$$B = \begin{pmatrix} 2 & 6 & 0 & 3 & 0 & 8 \\ 5 & 15 & 2 & 6 & 27 & 24 \\ -10 & -30 & 0 & -15 & -39 & -40 \\ -3 & -9 & 1 & -5 & -10 & -10 \end{pmatrix} \quad \text{and its} \quad ref(B) = \begin{pmatrix} 1 & 6 & 0 & 0 & 0 & 4 \\ 0 & 0 & 1 & 0 & 0 & 2 \\ 0 & 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 & 1 & 0 \end{pmatrix}$$

(a) A basis for row space of the matrix B is:

(b) A basis for column space of the matrix B is:

(c) The nullity of B is:

(d) The rank of B is:

(5) A matrix A is called *idempotent* if $A^2 = A$.

(a) For which values of k is the matrix $A = \begin{pmatrix} 2 & -2 & -4 \\ -1 & 3 & 4 \\ 1 & -2 & k \end{pmatrix}$ idempotent?

(b) For which value(s) of k $\det(A) = 0$?

(c) When $k = 3$ is A invertible? If yes use the cofactor method to find A^{-1}.

(6) Let

$$A = \begin{pmatrix} -7 & -6 \\ 12 & 10 \end{pmatrix}$$

(a) Find the eigenvalues of A.

(b) Find the eigenvectors of A.

(c) Explain whether A is diagonalizable or not. If yes find the diagonal matrix D.

(7) Consider the following linear transformation $T : \mathbb{R}^3 \to \mathbb{R}^3$ given by

$$T\begin{pmatrix} a \\ b \\ c \end{pmatrix} = \begin{pmatrix} a - 2c \\ 2a - 2b - 5c \\ -a + 2b + 8c \end{pmatrix}.$$

(a) Find the standard matrix representing T.

(b) Show that T is invertible.

(c) Find the standard matrix of T^{-1}.

(8) (Bonus) Consider the matrix

$$A = \begin{pmatrix} a & -b & -c & -d \\ b & a & d & -c \\ c & -d & a & b \\ d & c & -b & a \end{pmatrix}$$

(a) Compute $A^T A$.

(b) Use the above to prove that

$$\det(A) = (a^2 + b^2 + c^2 + d^2)^2$$