Polytechnic University

MA 1112/1412 — Exam 2 — May 6, 2005

Exam 1: ______

Exam 2: ______

Grade: ______

Print Name:

Signature:

ID #:

Instructor/Section:

Directions: You have **90 minutes** to answer the following questions. ***You must show all your work*** as neatly and clearly as possible and indicate the final answer clearly. You may use only a TI-30 calculator. The last few pages contain formulas that you might find useful. You may tear these pages out.

If you are feeling ill you should inform the proctor. The proctor will note your name, Poly ID and accept any written statement(s) that you may wish to make regarding your illness.

Problem	Possible	Points
1	15	
2	14	
3	14	
4	10	
5	12	
6	15	
7	10	
8	10	
Total	100	

YOUR SIGNATURE:

(1) Fill in the blanks. You do not need to explain.

(a) (Page 316, Problem 11) If we split the function $f(x) = \dfrac{11x+2}{x^2-x-6}$ into partial fractions of the form $\dfrac{A}{x+2} + \dfrac{B}{x-3}$,

then $A =$________________ and $B =$________________.

(b) (Page 298, Problem 72) Suppose that

$$\int_4^8 f(x)\ dx = 1.7 \quad \text{and} \quad \int_6^8 f(x)\ dx = 2.5.$$

Evaluate each of the following integrals.

(i) $\displaystyle\int_6^8 f(2x-8)\ dx =$ ________________

(ii) $\displaystyle\int_{\pi/6}^{\pi/2} f(8\sin x)\cos x\ dx =$ ________________

(2) Find each of the following integrals. You must show your work.

(a) (Page 308, Problem 31) $\displaystyle\int \frac{1}{x^2 - 6x + 13}\, dx$

(b) (Page 308, Problem 28) $\displaystyle\int_4^{e^2} \frac{1}{3z - z^2}\, dz$

(3) Find each of the following integrals. You must show your work.

(a) (Page 340, Problem 110) $\int_0^{\pi} \sin(3x)e^{-4x}\, dx$

(b) (Page 303, Problem 24) $\int \arctan(7z)\, dz$

(4) (Page 277, Problem 14) Fran throws a rock straight upward alongside a building (see figure). The rock rises until it is even with the top of the building and then falls back to the ground; it remains aloft (i.e. in the air) for 4 seconds. How tall is the building? (The acceleration due to gravity is $g = 9.8\text{m/s}^2$.)

(5) (Page 320, Problem 5) Fill in the blanks. You do not need to explain.

(a) The exact value of $\int_0^1 x^3 \, dx$ is __________________.

(b) If we use Mid(2) to estimate $\int_0^1 x^3 \, dx$, then Mid(2) = __________________.

(c) If we use Trap(2) to estimate $\int_0^1 x^3 \, dx$, then Trap(2) = __________________.

(6) (Page 277, Problems 1–10; Page 481, Problems 3–15) Fill in the blanks. You do not need to explain.

(a) The function $y(t) = \cos(3t)$ is a solution of the differential equation $y'' + ky = 0$,

if the constant $k =$__________________.

(b) The solution of the initial value problem $P'(t) = 2 + \sin(2t)$, when $P(0) = 5$ is

__________________.

(c) The function $y(t) = e^{x^3+cx}$ is a solution of the differential equation

$$y' = (3x^2 + 4)y,$$

if the constant $c =$__________________.

(7) (Page 304, Problem 50) Let f be a twice differentiable function, and define $F(t) = \int_1^t f(x)\,dx$. Some values of F, f and f' are given in the table below.

t	$F(t)$	$f(t)$	$f'(t)$
1	13	-3	2
2	3	-1	1.5
6	-1	-0.5	2.2

Evaluate the following integral. Show your work.

$$\int_2^6 x^2 f''(x)\,dx$$

(8) (Sample Exam) The table shows the daily mean temperature recorded during the first week of December at Big Frog, California. Estimate the **average** temperature during that period by using Midpoint approximation. Clearly indicate the number of subdivisions.

Days	12/1	12/2	12/3	12/4	12/5	12/6	12/7
Temperature (°C)	19	17.8	16.2	15.5	14	13	13.2

Useful formulas

- *Fundamental Theorem of Calculus*: If F is an antiderivative of f, then $\int_a^b f(x)\,dx = F(b) - F(a)$
- *The average value* of a function f on an interval $[a, b]$ is equal to $\frac{1}{b-a}\int_a^b f(x)\,dx$
- *Comparison of Definite Integrals*: If f is continuous and $m \le f(x) \le M$ for $a \le x \le b$, then $m(b-a) \le \int_a^b f(x)\,dx \le M(b-a)$
- The *acceleration* due to gravity, g:

  $$g = 9.81\text{m/sec}^2, \qquad \text{or} \qquad g = 32\text{ft/sec}^2$$
- *Integration by Parts*:

  $$\int u dv = uv - \int v du \qquad \text{or} \qquad \int uv' dx = uv - \int vu' dx$$
- *Numerical Approximations:*

  $$\text{TRAP}(n) = \frac{\text{LEFT}(n)+\text{RIGHT}(n)}{2}$$
- **Differentiation formulas**

$\frac{d}{dx}(x^n) = nx^{n-1}$	$\frac{d}{dx}(e^x) = e^x$	$\frac{d}{dx}(a^x) = (\ln a)a^x$
$\frac{d}{dx}(\ln\|x\|) = \frac{1}{x}$	$\frac{d}{dx}(\sin(x)) = \cos x$	$\frac{d}{dx}(\cos(x)) = -\sin x$
	$\frac{d}{dx}(\tan(x)) = \sec^2 x$	$\frac{d}{dx}(\cot(x)) = -\csc^2 x$
	$\frac{d}{dx}(\sec(x)) = \sec x \tan x$	$\frac{d}{dx}(\csc(x)) = -\csc x \cot x$
$\frac{d}{dx}(\arcsin(x)) = \frac{1}{\sqrt{1-x^2}}$	$\frac{d}{dx}(\arccos(x)) = \frac{-1}{\sqrt{1-x^2}}$	$\frac{d}{dx}(\arctan(x)) = \frac{1}{1+x^2}$

YOUR SIGNATURE:

Here a, b, c, d are constants.

A Short Table of Indefinite Integrals

I. Basic Functions

1. $\int x^n\ dx = \frac{1}{n+1}x^{n+1} + C,\ (n \neq -1)$
2. $\int \frac{1}{x}\ dx = \ln|x| + C$
3. $\int a^x\ dx = \frac{1}{\ln a}a^x + C$
4. $\int \ln x\ dx = x\ln x - x + C$
5. $\int \sin ax\ dx = -\frac{1}{a}\cos ax + C$
6. $\int \cos ax\ dx = \frac{1}{a}\sin ax + C$
7. $\int \tan ax\ dx = -\frac{1}{a}\ln|\cos ax| + C$

II. Products of e^x, $\cos x$, and $\sin x$

8. $\int e^{ax}\sin(bx)\ dx = \frac{1}{a^2+b^2}e^{ax}[a\sin(bx) - b\cos(bx)] + C$
9. $\int e^{ax}\cos(bx)\ dx = \frac{1}{a^2+b^2}e^{ax}[a\cos(bx) + b\sin(bx)] + C$
10. $\int \sin(ax)\sin(bx)\ dx = \frac{1}{b^2-a^2}[a\cos(ax)\sin(bx) - b\sin(ax)\cos(bx)] + C, \quad a \neq b$
11. $\int \cos(ax)\cos(bx)\ dx = \frac{1}{b^2-a^2}[b\cos(ax)\sin(bx) - a\sin(ax)\cos(bx)] + C, \quad a \neq b$
12. $\int \sin(ax)\cos(bx)\ dx = \frac{1}{b^2-a^2}[b\sin(ax)\sin(bx) + a\cos(ax)\cos(bx)] + C, \quad a \neq b$

III. Product of Polynomial $p(x)$ with $\ln x$,e^x, $\cos x$, and $\sin x$

13. $\int x^n \ln x\ dx = \frac{1}{n+1}x^{n+1}\ln x - \frac{1}{(n+1)^2}x^{n+1} + C, \quad n \neq -1, x > 0$
14. $\int p(x)e^{ax}\ dx = \frac{1}{a}p(x)e^{ax} - \frac{1}{a^2}p'(x)e^{ax} + \frac{1}{a^3}p''(x)e^{ax} - \cdots + C$

    $(+-+-+-+\ldots)$ (signs alternate)
15. $\int p(x)\sin ax\ dx = -\frac{1}{a}p(x)\cos(ax) + \frac{1}{a^2}p'(x)\sin(ax) + \frac{1}{a^3}p''(x)\cos(ax) - \cdots + C$

    $(-++--++--\ldots)$ (signs alternate in pairs)
16. $\int p(x)\cos ax\ dx = \frac{1}{a}p(x)\sin(ax) + \frac{1}{a^2}p'(x)\cos(ax) - \frac{1}{a^3}p''(x)\sin(ax) - \cdots + C$

    $(++--++--\ldots)$ (signs alternate in pairs)

IV. Integer Powers of $\sin x$ and $\cos x$

17. $\displaystyle\int \sin^n x\, dx = -\frac{1}{n}\sin^{n-1} x \cos x + \frac{n-1}{n}\int \sin^{n-2} x\, dx, \quad n \text{ positive}$

18. $\displaystyle\int \cos^n x\, dx = \frac{1}{n}\cos^{n-1} x \sin x + \frac{n-1}{n}\int \cos^{n-2} x\, dx, \quad n \text{ positive}$

19. $\displaystyle\int \frac{1}{\sin^m x}\, dx = -\frac{1}{m-1}\frac{\cos x}{\sin^{m-1} x} + \frac{m-2}{m-1}\int \frac{1}{\sin^{m-2} x}\, dx, \quad m \neq 1, m \text{ positive}$

20. $\displaystyle\int \frac{1}{\sin x}\, dx = \frac{1}{2}\ln\left|\frac{\cos x - 1}{\cos x + 1}\right| + C$

21. $\displaystyle\int \frac{1}{\cos^m x}\, dx = \frac{1}{m-1}\frac{\sin x}{\cos^{m-1} x} + \frac{m-2}{m-1}\int \frac{1}{\cos^{m-2} x}\, dx, \quad m \neq 1, m \text{ positive}$

22. $\displaystyle\int \frac{1}{\cos x}\, dx = \frac{1}{2}\ln\left|\frac{\sin x + 1}{\sin x - 1}\right| + C$

23. $\displaystyle\int \sin^m x \cos^n x\, dx :$
    If n is odd, let $w = \sin x$.
    If both m and n are even and non-negative, convert all to $\sin x$ or all to $\cos x$ (using $\sin^2 x + \cos^2 x = 1$), and use IV-17 or IV-18.
    If m and n are even and one of them is negative, convert to whichever function is in the denominator and use IV-19 or IV-21.
    The case in which both m and n are even and negative is omitted.

V. Quadratic in the Denominator

24. $\displaystyle\int \frac{1}{x^2 + a^2}\, dx = \frac{1}{a}\arctan\left(\frac{x}{a}\right) + C,\ a \neq 0$

25. $\displaystyle\int \frac{bx + c}{x^2 + a^2}\, dx = \frac{b}{2}\ln|x^2 + a^2| + \frac{c}{a}\arctan\left(\frac{x}{a}\right) + C,\ a \neq 0$

26. $\displaystyle\int \frac{1}{(x-a)(x-b)}\, dx = \frac{1}{(a-b)}(\ln|x-a| - \ln|x-b|) + C, \quad a \neq b$

27. $\displaystyle\int \frac{cx + d}{(x-a)(x-b)}\, dx = \frac{1}{(a-b)}[(ac+d)\ln|x-a| - (bc+d)\ln|x-b|] + C, \quad a \neq b$

VI. Integrands involving $\sqrt{a^2 + x^2}, \sqrt{a^2 - x^2}, \sqrt{x^2 - a^2}, a > 0$

28. $\displaystyle\int \frac{dx}{\sqrt{a^2 - x^2}} = \arcsin\left(\frac{x}{a}\right) + C$

29. $\displaystyle\int \frac{dx}{\sqrt{x^2 \pm a^2}} = \ln|x + \sqrt{x^2 \pm a^2}| + C$

30. $\displaystyle\int \sqrt{a^2 \pm x^2}\, dx = \frac{1}{2}\left(x\sqrt{a^2 \pm x^2} + a^2\int \frac{1}{\sqrt{a^2 \pm x^2}}\, dx\right) + C$

31. $\displaystyle\int \sqrt{x^2 - a^2}\, dx = \frac{1}{2}\left(x\sqrt{x^2 - a^2} - a^2\int \frac{1}{\sqrt{x^2 - a^2}}\, dx\right) + C$