Polytechnic University

MA 1022/1322 SAMPLE EXAM 1 FALL 2004

(1) (Page 112, Problem 57) At a time $t$ seconds after it is thrown up in the air, a tomato is at a height of $h(t) = -4.9t^2 + 25t + 3$ meters.

Fill in the blanks. You do not need to show your work. Keep your answer accurate to 2 decimal places.

(a) The velocity of the tomato at $t = 2$ is ____________ meters/sec.

(b) The acceleration of the tomato at $t = 2$ is ____________ meters/sec$^2$.

(c) The tomato is in the air for ____________ seconds.

(d) The highest point the tomato reaches is ____________ meters from the ground.

(e) The average velocity of the tomato during the time it goes up is ____________ meters/sec.

(2) (Page 159, Problems 1–52) Find derivatives for the functions given below. Assume $a$, $b$, $c$, and $k$ are positive constants. You must simplify your answers.

(a) $g(x) = \dfrac{x^2 + \sqrt{x} + 1}{x^{3/2}}$

(b) $g(w) = \dfrac{1}{2^w + e^w}$

(c) $y = \arctan\left(\dfrac{2}{x}\right)$

(d) $y = (x^2 + 5)^3(3x^3 - 2)^2$

(e) $s(y) = \sqrt[3]{\cos^2(y) + 3 + \sin^2(y)}$ (Hint: Simplify the function)

(f) $f(x) = \dfrac{a^2 - x^2}{a^2 + x^2}$

(g) $f(t) = (\sin(2t) - \cos(3t))^4$

(h) $m(x) = \ln\left(\dfrac{1 - \cos(t)}{1 + \cos(t)}\right)^4$ (Hint: Simplify the function)

(i) $h(w) = w \arcsin(w)$

(j) $g(t) = t\cos(\sqrt{t}\, e^t)$

(k) $y(t) = \sqrt{at} + a\sqrt{t} + \dfrac{a}{\sqrt{t}} - (\sqrt{a})\, t + \sqrt{a}$

(l) $y(x) = \tan(x^k) + \dfrac{1}{\sin(x + e^k)}$

(3) (Trig Homework) An airplane flies from Ft. Dickson to Tremont, a distance of 263 miles, and then turns through an angle of 55° and flies to Oroville, a distance of 135 miles. The pilot then wants to fly back to Ft. Dickson. Through what angle must he turn at Oroville to fly back to Ft. Dickson by the shortest route AND how far is it from Oroville to Ft. Dickson? You must show all of your work.

The angle is ____________.

The distance is ____________.

(4) (Trig Homework) Express in terms of $x$ without using trigonometric functions. Here $0 < x < 1$.

(a) $\tan(2\cos^{-1}(x))$

(b) $\sin(2\arctan(x))$

(5) The following table gives some of the values of $g(x)$ $g'(x)$, $f(x)$ and $f'(x)$:

| $x$ | $g(x)$ | $g'(x)$ | $f(x)$ | $f'(x)$ |
|---|---|---|---|---|
| $-2$ | $-3$ | $1/2$ | $-2$ | $-1$ |
| $1/4$ | $1$ | $7$ | $2$ | $-3$ |
| $1$ | $3$ | $2$ | $4$ | $-1$ |
| $4$ | $2$ | $-3$ | $1$ | $-2$ |

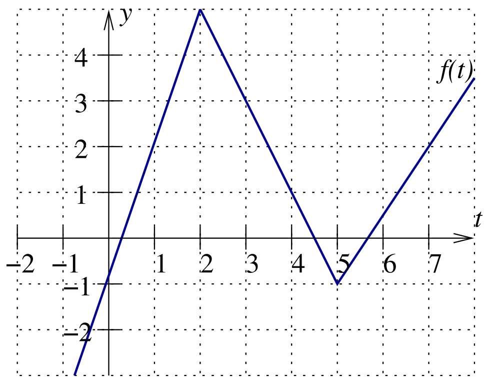
Fill in the blanks. You do not need to show your work.

(a) Based on the data given in the table and using tangent line approximation, the best possible estimate for $f(-1.95)$ is ______________.

(b) The derivative of $f(x)g(x/4)$ at $x = 1$ is ______________.

(c) The derivative of $\sqrt{f^2(x) + g^2(x)}$ at $x = 4$ is ______________.

(6) The graph of $f$ is drawn below.




(a) Find $g'(1)$ if $g(t) = \dfrac{f(t) - f(1)}{t+1}$ .

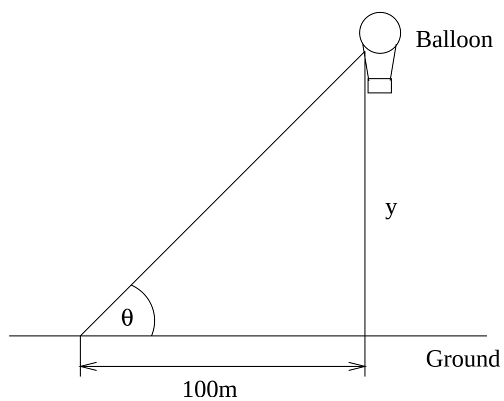
(b) Find $h'(6)$ if $h(t) = \cos\big(\pi f(t)\big) + f\big(\cos(\pi t)\big)$.

(7) (Page 126, Problem 51) Find the intervals where the function

$$h(z) = (e^z - 1)^2$$

is concave up. Show your work.

(8) A weather balloon that is rising vertically is observed from a point on the ground 100 m from the spot directly beneath the balloon. How fast is the balloon rising when the angle between the ground and the observer's line of sight is 45° and is increasing at 1° per second? Include units in your answer.



**Useful formulas**

For a triangle with sides $a$, $b$, $c$ and angles $A$, $B$, $C$ opposite these sides, respectively.

- Law of Sines:
$$\frac{\sin A}{a} = \frac{\sin B}{b} = \frac{\sin C}{c}$$
- Law of Cosines:
$$c^2 = a^2 + b^2 - 2ab\cos C$$
- Double angle:
$$\sin(2t) = 2\sin t\cos t \qquad \cos(2t) = \cos^2 t - \sin^2 t$$
- Sum-/Difference-of-angle:
$$\sin(\theta + \phi) = \sin(\theta)\cos(\phi) + \sin(\phi)\cos(\theta)$$
$$\sin(\theta - \phi) = \sin(\theta)\cos(\phi) - \sin(\phi)\cos(\theta)$$
$$\cos(\theta + \phi) = \cos(\theta)\cos(\phi) - \sin(\theta)\sin(\phi)$$
$$\cos(\theta - \phi) = \cos(\theta)\cos(\phi) + \sin(\theta)\sin(\phi)$$
- Vertex form of a quadratic function:
$$y = a(x - h)^2 + k$$

## Formulas you might find useful

- **The derivative of a function**

$$f'(x) = \lim_{h\to 0} \frac{f(x+h)-f(x)}{h}$$

- **Some rules of differentiation**

$$\frac{d}{dx}(cf(x)) = cf'(x)$$
$$\frac{d}{dx}(f(x)g(x)) = f'(x)g(x) + f(x)g'(x)$$
$$\frac{d}{dx}\left(\frac{f(x)}{g(x)}\right) = \frac{f'(x)g(x) - f(x)g'(x)}{g(x)^2}$$
$$\frac{d}{dx}f(g(x)) = f'(g(x))g'(x)$$

- **Differentiation formulas**

| | | |
|---|---|---|
| $\frac{d}{dx}(x^n) = nx^{n-1}$ | $\frac{d}{dx}(e^x) = e^x$ | $\frac{d}{dx}(a^x) = (\ln a)a^x$ |
| $\frac{d}{dx}(\ln x) = \frac{1}{x}$ | $\frac{d}{dx}(\sin(x)) = \cos x$ | $\frac{d}{dx}(\cos(x)) = -\sin x$ |
| | $\frac{d}{dx}(\tan(x)) = \sec^2 x$ | $\frac{d}{dx}(\cot(x)) = -\csc^2 x$ |
| | $\frac{d}{dx}(\sec(x)) = \sec x \tan x$ | $\frac{d}{dx}(\csc(x)) = -\csc x \cot x$ |
| $\frac{d}{dx}(\arcsin(x)) = \frac{1}{\sqrt{1-x^2}}$ | $\frac{d}{dx}(\arccos(x)) = \frac{-1}{\sqrt{1-x^2}}$ | $\frac{d}{dx}(\arctan(x)) = \frac{1}{1+x^2}$ |
| $\frac{d}{dx}(\sinh(x)) = \cosh(x)$ | $\frac{d}{dx}(\cosh(x)) = \sinh(x)$ | $\frac{d}{dx}(\tanh(x)) = \frac{1}{\cosh^2(x)}$ |

- **The linear approximation** of a function $f$ at $a$ is given by

$$y = f(a) + f'(a)(x-a)$$

- **Geometry Formulas**
  Here $V$ is the volume, $h$ is the height and $r$ is the radius.
  Cylinder: $V = \pi r^2 h$
  Cone: $V = \frac{1}{3}\pi r^2 h$
  Sphere: $V = \frac{4}{3}\pi r^3$